

**A Computer-Aided Engineering method and apparatus for predicting a quantitative value of a physical property at a point from waves generated by or scattered from a body.**

5 FIELD OF THE INVENTION

This invention relates to Computer-Aided Engineering (CAE) and in particular to a Computer-Aided Engineering (CAE) method and apparatus for predicting a quantitative value of a physical property at a point from waves generated by or scattered from a body. For example, the physical property can  
10 be an acoustic signature of a vibrating structure.

BACKGROUND OF THE INVENTION

The acoustic performance of manufactured products is becoming an extremely important aspect in the design and development process, not only  
15 to improve the comfort of the user of the product (e.g. passengers in a car, in an aircraft), but also to reduce the nuisance to the surroundings (e.g. habitations close to highways or to an airport). A typical problem may be described or represented for numerical analysis as a vibrating body surrounded by a fluid which may be assumed effectively infinite at least as an  
20 approximation.

The concept of Acoustic Transfer Vectors (ATV's) is known [Y. K. Zhang, M.-R. Lee, P. J. Stanecki, G. M. Brown, T. E. Allen, J. W. Forbes, Z. H. Jia, "Vehicle Noise and Weight Reduction Using Panel Acoustic Contribution Analysis", SAE95, Paper 95NV69].

25 ATV's are input-output relations between the normal structural velocity of the vibrating surface and the sound pressure level at a specific field point (see figure 3). ATV's can be interpreted as an ensemble of Acoustic Transfer Functions from the surface nodes to a single field point. As such ATV's can be measured and are a physical parameter of a system. Literature refers  
30 sometimes to this concept as Contribution Vectors or Acoustic Sensitivities.

ATV's only depend on the configuration of the acoustic domain, i.e. the

shape of the vibrating body and the fluid properties controlling the sound propagation (speed of sound and density), the acoustic surface treatment, the frequency and the field point. They do not depend on the loading condition. The concept of ATV's, and their properties, is not new, and has already been published in several scientific papers.

There are two computer-aided engineering processes that are key for evaluating and optimising the acoustic performance of structures: the ability to predict the acoustic radiation pattern of a vibrating structure, either from computed or measured surface vibrations (acoustic radiation prediction), and the ability to recover surface vibrations onto a vibrating structure from measured field sound pressure level. The latter one is sometimes the only manner to perform source identification, when it is impossible to apply measurement devices onto the structure surface (e.g. rotating tire).

These engineering processes rely on numerical analysis methods, amongst which the most popular are currently the various forms of the boundary element method (the direct mono-domain, the direct multi-domain and the indirect approaches), and the finite element method, that can be extended to handle unbounded regions, e.g. using infinite elements. Numerical analysis is well known to the skilled person, e.g. "The finite element method", Zienkiewicz and Taylor, Butterworth-Heinemann, 2000; "Numerical Analysis", Burden and Faires, Brooks/Cole, 2001; H. A. Schenck, "Improved integral formulation for acoustics radiation problems", J. Acoust. Soc. Am., 44, 41-58 (1981); A. D. Pierce, "Variational formulations in acoustic radiation and scattering", Physical Acoustics, XXII, 1993.; J. P. Coyette, K. R. Fyfe, "Solutions of elasto-acoustic problems using a variational finite element/boundary element technique", Proc. of Winter Annual Meeting of ASME, San Francisco, 15-25 (1989).

US 5,604,893 and US 5,604,891 describe finite element methods for solving acoustic problems by using an oblate finite element. The oblate finite element is based on a multipole expansion that describes a scattered or radiated field exterior to an oblate spheroid.

The computational cost associated with numerical methods for acoustic performance prediction is usually very large, and is linearly proportional to:

- the number of operational conditions (may be about a 100 different cases);
- 5 • the number of frequency lines to be evaluated for obtaining a representative response function (will typically be about 100 to 200);
- the number of design variants to be studied.

#### SUMMARY OF THE INVENTION

10 It is an object of the present invention to provide a method and apparatus for analysing physical systems by wave theory which is more efficient.

It is also an object of the present invention to provide a method and apparatus for analysing physical systems by wave theory which is quicker to  
15 carry out.

It is also an object of the present invention to provide a method and apparatus for analysing a physical system by wave theory which allows rapid recalculation following design modifications to the system.

The present invention provides a method for computing a Wave  
20 Transfer Vector based on the reciprocity principle, comprising the steps of:  
simulating positioning of a monopole, omnidirectional wave energy source at a reference position remote from a body;  
computing a boundary oscillation amplitude of the wave generated by the source at a surface of the body;  
25 deriving from the boundary oscillation amplitude said Wave Transfer Vector. A wave transfer vector of a body or structure surrounded by a medium may be described by the following equation:

$$\text{Prop} = \{w_{tv}\}^T \{B_c\}$$

where Prop is a physical property to be determined at a field point remote from  
30 a body or structure whereby the oscillatory surface boundary conditions of the body or structure are defined as  $B_c$ . The surface boundary conditions

generate or scatter waves which propagate through the medium to the field point and determine the physical property there. The wave transfer vector may be described as an array of transfer functions between the surface boundary conditions of a body or structure and a physical property of the medium generated at a field point. The medium does not have to uniform, i.e. made of a uniform materials. The field point may be in an internal domain (within the structure) and/or in an external domain (outside the structure).

The method may be used to analyze a structure or body in a medium in which Helmholtz's equation applies to the propagation of waves in the medium. The oscillatory surface states of the structure or body form one set of boundary conditions for the solution of Helmholtz's equation, e.g. at a field point.

The computing step may be carried out by a numerical method, which may comprise one of: a finite element method; a combination of finite and infinite element methods; a direct boundary element method; a direct multi-domain boundary element method; and an indirect boundary element method. The numerical method may be carried out on a digital computing system. The wave source may comprise an acoustic source.

The invention also provides a method for computing an additional Wave Transfer Vector comprising the steps of:

computing at least a first and a second wave transfer vector at a first and a second predetermined frequency, respectively,

computing the additional Wave Transfer Vector at a frequency intermediate the first and second frequency by interpolation between the first and second Wave Transfer Vectors.

A Wave Transfer Vector may be calculated or measured. The interpolation technique may comprise, for example, a polynomial interpolation mechanism (e.g. linear) or a spline interpolation mechanism (e.g. cubic spline) or any other suitable interpolation technique.

The invention also provides a method to compute a Modal Acoustic Transfer Vector (MATV) from an acoustic transfer vector (ATV) for a

projecting the ATV into the alternative co-ordinate system. The set of shapes may be defined by eigen-frequencies (natural frequencies or natural modes) of the body or by Ritz vectors (which may be described as operational modes of the system generated by applying a specific operational state), for example. The MATV may be used to predict a response of the body or the effect of such a response at a reference point remote from the body.

The present invention also includes methods and apparatus which combines two or more of ATV calculation using the reciprocity principle, generation of additional ATV's by interpolation, solution of Helmholtz's equation using MATV derived from ATV's by projecting ATV's into modal space and generation of additional MATV's by interpolating between ATV's.

One or more methods of the invention may support an acoustic  
15 radiation prediction engineering process, in particular but not limited to  
automotive engine and powertrain noise radiation application.

One or more methods of the invention may support a modal-based acoustic radiation prediction engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

One or more methods of the invention may support an inverse numerical acoustics engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

One or more methods of the invention may support a modal-based inverse numerical acoustics engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

Shapes combined to build a structural response during a method of the invention may comprise Ritz Vectors.

One or more methods of the invention may support a Ritz-vectors-based inverse numerical acoustics engineering process, in particular but not limited to automotive engine and powertrain noise radiation application.

One or more methods of the invention may support an evaluation of

multiple design alternatives, with respect to the acoustic performance prediction of vibrating structures.

One or more methods of the invention may support an evaluation of multiple design alternatives, with respect to the modal-based acoustic performance prediction of vibrating structures.

One or more methods of the invention may support an automated structural-acoustic optimization process, with the objective function being the acoustic performance prediction of vibrating structures.

One or more methods of the invention may support an automated structural-acoustic optimization process, with the objective function being the modal-based acoustic performance prediction of vibrating structures.

The invention also provides a processing engine adapted to carry out any of the methods of the invention.

The invention also provides a computer program product for execution on a computer, the computer program product executing any of the method steps of the invention.

The invention also provides a method of inputting at a near terminal a representation of a body and coordinates of a reference point and transmitting these to a remote terminal running a program for executing any of the method steps of the invention, and receiving at a near location an output of any of the methods. The output may comprise one of: an ATV, an oscillation amplitude such as an acoustic pressure level, a surface vibration of the body, a revised design of at least a part of the body, an acoustic intensity at a field point, a radiated acoustic power, a radiation efficiency, for example. Generally, the output may be a physical value at a field point or an integral or differential thereof, e.g. of an electric or magnetic field.

The present invention also comprises a computer system having a memory for computing a Wave Transfer Vector based on the reciprocity principle, comprising:

means for simulating positioning of a monopole, omnidirectional wave energy source at a reference position remote from a body;

means for computing a boundary oscillation amplitude of the wave generated by the source at a surface of the body; and  
means for deriving from the boundary oscillation amplitude said Wave Transfer Vector. The Wave Transfer Vector may be an Acoustic Transfer Vector.

5       The invention consists in one aspect in a new methodology to dramatically speed up the evaluation of the Wave Transfer Vectors, especially Acoustic Transfer Vectors and their exploitation. In particular, the use of the reciprocity principle provides improved accuracy of the ATV's because they can be computed and stored on an element basis and provides a reduction in  
10       computational effort, since the solution needs to be performed for a number of excitation vectors equal to the number of field points of interest, instead of to the number of boundary nodes.

Further on, the use of efficient interpolation techniques is very appropriate for evaluating ATV's, due to their smooth frequency dependence,  
15       and allows the restriction of their evaluation to a restricted set of master frequency lines, which are then de-coupled from the excitation frequency contents. This results in significant gains in computing time and data storage space. The use of ATV's allows a practical design optimization scheme because the computer intensive task (calculation of the ATV's) is performed  
20       up front, outside of the optimization loop (Fig. 9a and b).

The developed methodology results therefore in a reduction of the computational cost attached to acoustic performance prediction and wave theory problems in general, making it quasi insensitive to the number of operational conditions, to the number of frequency lines of interest, and to the  
25       number of design variants. As such, it allows the acoustic performance prediction tool to be driven by an automated design exploration and optimisation tool.

The present invention will now be described by way of example only and with reference to the accompanying drawings.

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## BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is a schematic representation of Acoustic radiation Prediction;

Figure 2 is a schematic diagram of Inverse Numerical Acoustics;

Figure 3 is a schematic diagram of the concept of Acoustic Transfer Vectors;

5        Figure 4 is a schematic diagram of a direct method for evaluating Acoustic Transfer Vectors. This has to be repeated  $n$  times.

Figure 5 is a schematic diagram of a reciprocal method for evaluating Acoustic Transfer Vectors in accordance with an embodiment of the present invention;

10        Figure 6 is representation of a linear interpolation of an Acoustic Transfer Vector in accordance with an embodiment of the present invention. The interpolation curve (B) is superimposed on the ATV curve (R);

Figure 7 shows a spline interpolation of an Acoustic Transfer Vector in accordance with an embodiment of the present invention. The interpolation curve (B) is superimposed on the ATV curve (R);

15        Figure 8 shows an example of panel acoustic contribution analysis;

Figures 9A and B show flow charts of an optimization process based on ATV's and MATV's respectively, in accordance with embodiments of the present invention;

20        Figure 10 is a block diagram of an apparatus useful for performing methods of the invention; and

Figure 11 is a representation of a computer system for use with the present invention.

## 25    DEFINITIONS

Acoustic radiation prediction: is an engineering process that aims at predicting the sound pressure level in the surrounding field of a vibrating structure (see figure 1). Surface vibrations create acoustic pressure waves that propagate to the receiver.

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Inverse numerical acoustics: is an engineering process that aims at deriving



the surface vibration velocity from field sound pressure measurements (see figure 2). As such, it is the inverse problem to acoustic radiation prediction.

Discretization Methods: in order to solve acoustic radiation prediction problems and inverse numerical acoustics problems, computer models are built using numerical methods. The most frequently used numerical methods are the boundary element method (BEM) and the finite-infinite element method (FEM/IFEM). These methods rely upon a discretization of the geometrical domain (FEM/IFEM) or of its boundary (BEM) and solve, numerically, the wave equation in the frequency domain. There exists substantial published literature concerning both theoretical and practical aspects of these numerical methods, for some examples of which please see the bibliography included in this specification. The major computational effort involved within the application of many such numerical methods concerns the solution of a large system of linear equations.

#### DETAILED DESCRIPTION OF THE ILLUSTRATIVE EMBODIMENTS

The present invention will mainly be described with reference to acoustic waves but the invention is not limited thereto. The concepts underlying the present invention may be applied to any vibrating or oscillating energy wave which can be described by wave equations. For example, the present invention may be applied to electromagnetic waves. The present invention preferably relies on any suitable numerical methods for solving such equations and problems, such as the various forms of the boundary element method (the direct mono-domain, the direct multi-domain and the indirect approaches), and the finite element method, that can be extended to handle unbounded regions, e.g. using infinite elements.

Generally, the methods of the present invention may be applied to a body or structure surrounded by (external domain) or containing (internal domain) a medium in which waves, generated or scattered by surface states of the structure or body are propagated. The medium does not have to be

uniform. One example of such a physical system is a vibrating structure surrounded by a medium such as air. Acoustic performance prediction is applied to many different kinds of products, in a process that is well standardised. As an example, let us consider an automotive powertrain structure, whose goal is to minimise the acoustic sound emission, and try to quantify the benefit due to the invention.

The following is a typical practical problem. A reasonable boundary element model will include about 7000 nodes/elements. It should be analysed under 50 different operating conditions, corresponding to different engine speed (RPM's). For building a full frequency response, about 200 frequency lines need to be computed. The performance at 10 microphone locations is of interest, for example to evaluate the radiated sound power (according to the ISO3744-1981 procedure).

The computational cost for processing such a model on an engineering workstation is based on following figures: the matrix assembly and factorisation process takes 5,400 seconds per frequency, and the back-substitution process (needed for each excitation case) takes 3 seconds per frequency. Assuming that about 20 different design alternatives need to be analysed in the design exploration process, the following estimations are obtained:

- Using the traditional acoustic prediction technique (see Figure 1) the total computational effort can be estimated to be the number of frequency lines (200) times the time for a single frequency analysis ( $5,400 \text{ sec} + 3 \text{ sec} \times 50$  load cases), times the number of design alternatives (20), i.e. 22.2 million seconds, corresponding to 257 computing days

Using the Acoustic Transfer Vectors technique (Figures 3 and 4), computed in a direct way, the model has to be solved for a number of excitation cases equal to the number of nodes (7000), but since the exploitation time (scalar product) is neglectible, the total computational cost is independent of the number of design alternatives, and of operating conditions. Therefore, the total

computational effort is estimated to be the number of frequency lines (200) times the time for a single frequency analysis (5,400 sec + 3 sec x 7000 right-hand-side vectors) i.e. 5.28 million seconds, corresponding to 61 computing days.

- 5 • Using an embodiment of the present invention, i.e. an Acoustic Transfer Vectors technique computed in the reciprocal manner (Figure 5 onwards) combined with an interpolation technique with 10 master frequencies, and neglecting the time for ATV interpolation and exploitation, the total computational effort can be estimated to be the number of master frequency
- 10 lines (10) times the time for a single frequency analysis (5,400 sec + 3 sec x 10 load cases), i.e. 54,300 seconds, corresponding to 0.6 computing days.

This example shows that, given the above assumptions, the present invention leads to a reduction of the computational effort by a factor of 100 compared to the conventional ATV approach, and 400 compared to the

15 traditional acoustic radiation prediction technique.

### Theoretical background

#### *Notations*

$A, B, H, Q$	Fluid domain matrix operators
$ATV$	Acoustic Transfer Vector
$ATM$	Acoustic Transfer Matrix
$C$	Sound velocity
$C$	Coupling matrix
$G$	Green's function
$F$	Frequency
$K$	Wave number
$K, D, M$	Fluid stiffness, damping and mass matrix operators
$J$	Imaginary number
$N$	Surface normal
$P$	Acoustic pressure
$p_b$	Boundary acoustic pressure

$S$	Surface of the vibrating (emitting) structure
$U, V$	Complex matrices (SVD)
$V$	Acoustic particle velocity
$v_b$	Boundary structural velocity
$v_s$	Structural velocity
$x$	Observation point
$y$	Source point
$\beta$	acoustic admittance (complex)
$\rho$	mass density (complex)
$\sigma$	Single layer potential; singular values
$\mu$	Double layer potential
$\omega$	Circular frequency
$i$	Internal degrees of freedom (superscript)
$j$	Interface degrees of freedom (superscript)

### *Governing Equation*

Acoustic wave propagation in bounded or unbounded regions is governed by the wave equation. Assuming an harmonic behaviour, the time and space variables may be separated, leading to Helmholtz' equation:

$$\nabla^2 p + k^2 p = 0$$

where  $k$  is the wave number  $\omega/c$ .

Four numerical formulations are typically available for solving Helmholtz equation over a region bounded by a surface  $S$ , with proper boundary conditions, and to build Acoustic Transfer Vectors in an improved way following the principles of the invention. Each of these methods may be implemented on a digital computing system by the appropriate software complied to execute on the computing system, especially by using numerical analysis.

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1. The direct boundary element method (DBEM) formulation, using the acoustic pressure and velocity as boundary variables. This formulation can be

used to model homogeneous interior or exterior domains.

2. The multi-domain direct boundary element method (MDDBEM) formulation, using the acoustic pressure and velocity as boundary variables.
- 5 This formulation can be used to model a non-homogeneous domain by dividing it into a set of homogeneous sub-domains that are linked together by continuity conditions.
3. The indirect boundary element method (IBEM) formulation, using the single and double layer potentials (acoustic velocity jump and pressure jump respectively) as boundary variables. This formulation can be used to model a homogeneous domain, where both sides of the boundary radiate. In particular, it can be used to model an interior domain, an exterior domain or both simultaneously.
- 15 4. The acoustic finite/infinite element method (FEM) formulation, using the acoustic pressure as boundary variables. This formulation can be used to model a non-homogeneous interior domain, an exterior domain or both simultaneously.

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#### *Direct Boundary Element Method (DBEM) Formulation*

Using the DBEM formulation, the acoustic pressure at any point of a homogeneous fluid domain containing no acoustic source can be expressed in terms of the acoustic pressure on the boundary domain and its normal derivative [see H. A. Schenk, "Improved integral formulation for acoustics radiation problems", J. Acoust. Soc. AM. 44, 41-58 (1981)]:

25

$$p(\vec{x}) = \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y - \int_S \frac{\partial p(\vec{y})}{\partial n_y} G(\vec{x}|\vec{y}) dS_y \quad (1)$$

where  $p(\vec{y})$  is the acoustic pressure on the boundary and  $\frac{\partial p(\vec{y})}{\partial n_y}$  its normal derivative,  $\vec{n}_y$  is the inward normal at point  $\vec{y}$  on the boundary and  $G(\vec{x}|\vec{y})$  is

the Green's function. Making use of the Euler equation, equation (1) becomes:

$$p(\vec{x}) = \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega \int_S v(\vec{y}) G(\vec{x}|\vec{y}) dS_y \quad (2)$$

where  $v(\vec{y})$  is the normal acoustic velocity on the boundary,  $\rho$  is the fluid mass density and  $\omega$  is the angular frequency. Note that the boundary acoustic velocity is related to the structural velocity through the following relationship:

$$v(\vec{y}) = v_s(\vec{y}) + \beta(\vec{y})p(\vec{y}) \quad (3)$$

where  $v_s(\vec{y})$  is the structural velocity and  $\beta(\vec{y})$  the boundary admittance.

Using equation (3), equation (2) becomes

$$p(\vec{x}) = \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega \int_S \beta(\vec{y}) p(\vec{y}) G(\vec{x}|\vec{y}) dS_y + j\rho\omega \int_S v_s(\vec{y}) G(\vec{x}|\vec{y}) dS_y \quad (4)$$

This equation is true in the domain and on its boundary. Nevertheless, when evaluated on the boundary, the Green's function and its normal derivative become singular. Whereas the last two integrals of equation (4) are regular, the first one is singular and should be evaluated in the Cauchy's principal value sense:

$$c(\vec{x})p(\vec{x}) = P.V. \int_S p(\vec{y}) \frac{\partial G(\vec{x}|\vec{y})}{\partial n_y} dS_y + j\rho\omega \int_S \beta(\vec{y}) p(\vec{y}) G(\vec{x}|\vec{y}) dS_y + j\rho\omega \int_S v_s(\vec{y}) G(\vec{x}|\vec{y}) dS_y \quad (5)$$

where

$$c(\vec{x}) = 1 + P.V. \int_S \frac{1}{4\pi|\vec{x} - \vec{y}|^2} \frac{\partial |\vec{x} - \vec{y}|}{\partial n_y} dS_y \quad (6)$$

in the three dimensional space. Note that  $c(\vec{x}) = 1/2$  for a smooth surface around  $\vec{x}$ .

Once discretized using boundary elements and evaluated at the mesh nodes, equation (5) leads to the following matrix system:

$$[A]\{p_b\} = [B]\{v_b\} \quad (7)$$

where the subscript  $b$  stands for *boundary*. Similarly, equation (4) gives:

$$p = \{d\}^T \{p_b\} + \{m\}^T \{v_b\} \quad (8)$$

Combining equations (7) and (8) leads to:

$$p = \{atv\}^T \{v_b\} \quad (9)$$

where  $\{atv\}$  is the Acoustic Transfer Vector, given by:

$$\{atv\}^T = \{d\}^T [A]^{-1} [B] + \{m\}^T \quad (10)$$

The Acoustic Transfer Vector (ATV) is therefore an array of transfer functions between the surface normal velocity and the pressure at the field point. Finally, when the pressure is evaluated at several locations, equation (9) can be rewritten as:

$$\{p\} = [ATM]^T \{v_b\} \quad (11)$$

where the Acoustic Transfer Matrix  $[ATM]$  is formed by the different Acoustic Transfer Vectors.

More generally, a generalised wave transfer vector of a body or structure surrounded by a medium (external domain) or containing a medium (internal domain) may be described by the following equation:

$$Prop = \{wtv\}^T \{Bc\}$$

where  $Prop$  is a quantitative value of a physical property to be determined at a field point remote from a body or structure whereby the oscillatory surface boundary conditions of the body or structure are defined as  $Bc$ . The boundary conditions may be Dirichlet, Neumann or Robin boundary conditions. The surface boundary conditions generate or scatter waves which propagate through the medium to the field point and determine the physical property there. The wave transfer vector may be described as an array of transfer functions between the surface boundary conditions of a body or structure and a physical property of the medium generated at a field point. The waves may be any waves which can be described by Helmholtz's equation, e.g. acoustic waves or, for example, electromagnetic waves scattered from an object or generated by an antenna. The medium surrounding the body does not have to be uniform.

The principles of Wave Transfer Vectors and ATV's described above may be applied to any of the further embodiments described below.

### *Multi-Domain Direct Boundary Element Method (MDDBEM) Formulation*

- 5 In case the acoustic region includes partitions characterised with different fluid material properties, the above formulation cannot be directly used since it is only valid for homogeneous domains. In such cases, a multi-domain integral formulation has to be used. The global acoustic region is then decomposed into sub-domains with the requirement that, within a sub-domain, 10 the fluid properties need to be homogeneous. At the interface between the sub-domains, continuity conditions are enforced.

For sake of simplicity, let's consider here a two-domain model. Equation (7) for the first sub-domain can be written as:

$$\begin{bmatrix} A_1^{ii} & A_1^{ij} \\ A_1^{ji} & A_1^{jj} \end{bmatrix} \begin{Bmatrix} p_1^i \\ p_1^j \end{Bmatrix} = \begin{bmatrix} B_1^{ii} & B_1^{ij} \\ B_1^{ji} & B_1^{jj} \end{bmatrix} \begin{Bmatrix} v_1^i \\ v_1^j \end{Bmatrix} \quad (12)$$

- 15 where superscript  $i$  stands for internal degrees of freedom and superscript  $j$  stands for interface degrees of freedom. Similarly, equation (7) can be rewritten for the second sub-domain as:

$$\begin{bmatrix} A_2^{ii} & A_2^{ij} \\ A_2^{ji} & A_2^{jj} \end{bmatrix} \begin{Bmatrix} p_2^i \\ p_2^j \end{Bmatrix} = \begin{bmatrix} B_2^{ii} & B_2^{ij} \\ B_2^{ji} & B_2^{jj} \end{bmatrix} \begin{Bmatrix} v_2^i \\ v_2^j \end{Bmatrix} \quad (13)$$

- The continuity of the normal velocity and acoustic pressure has to be satisfied 20 at the interface between of the two sub-domains:

$$\begin{cases} v_1^j = -v_2^j = v^j \\ p_1^j = p_2^j = p^j \end{cases} \quad (14)$$

Combining equations (12), (13) and (14) leads to the global system of equations:

$$\begin{bmatrix} A_1^{ii} & A_1^{ij} & -B_1^{ij} & 0 \\ A_1^{ji} & A_1^{jj} & -B_1^{jj} & 0 \\ 0 & A_2^{ij} & B_2^{ij} & A_2^{ii} \\ 0 & A_2^{ji} & B_2^{ji} & A_2^{jj} \end{bmatrix} \begin{Bmatrix} p_1^i \\ p_1^j \\ v^j \\ p_2^i \end{Bmatrix} = \begin{bmatrix} B_1^{ii} & 0 \\ B_1^{ji} & 0 \\ 0 & B_2^{ii} \\ 0 & B_2^{ji} \end{bmatrix} \begin{Bmatrix} v_1^i \\ v_2^i \end{Bmatrix} \quad (15)$$

- 25 Using equation (15), one can define the Acoustic Transfer Vectors for the



global system:

$$p = \{atv\}^T \begin{Bmatrix} v_1^i \\ v_2^i \end{Bmatrix} \quad (16)$$

and similarly derive the Acoustic Transfer Matrix  $[ATM]$  when the acoustic pressure has to be evaluated at different locations.

5

### *Indirect Boundary Element Method (IBEM) Formulation*

Using an indirect formulation, the acoustic pressure at any point of the domain can be expressed in terms of single and double layer potentials [see A. D. Pierce, "Variational formulations in acoustic radiation and scattering",

10 Physical Acoustics XXII, 1993]:

$$p(\bar{x}) = \int_S \mu(\bar{y}) \frac{\partial G(\bar{x}|\bar{y})}{\partial n_y} dS_y - \int_S \sigma(\bar{y}) G(\bar{x}|\bar{y}) dS_y \quad (17)$$

where  $\sigma(\bar{y})$  and  $\mu(\bar{y})$  are the single and double layer potentials, respectively.

For the Neuman problem (i.e. structural velocity boundary condition) and when both sides of the boundary have the same velocity (i.e. thin shell),

15 this equation can be expressed only in terms of double layer potentials:

$$p(\bar{x}) = \int_S \mu(\bar{y}) \frac{\partial G(\bar{x}|\bar{y})}{\partial n_y} dS_y \quad (18)$$

Equation (18) can be evaluated on an arbitrary surface  $S'$ . It can be derived with respect to the normal at  $x \in S'$ , pre-multiplied by an admissible test function  $\delta\mu(x)$  and integrated over the surface  $S'$ :

$$20 \quad \int_{S'} \frac{\partial p(\bar{x})}{\partial n_x} \delta\mu(\bar{x}) dS' = \int_{S'} \int_S \mu(\bar{y}) \frac{\partial^2 G(\bar{x}|\bar{y})}{\partial n_x \partial n_y} \delta\mu(\bar{x}) dS_y dS'_x \quad (19)$$

Now, let the surface  $S'$  tend to  $S$  and make use of the Euler equation:

$$- \int_S j\omega\rho v(\bar{x}) \delta\mu(\bar{x}) dS = \int_S \int_S \mu(\bar{y}) \frac{\partial^2 G(\bar{x}|\bar{y})}{\partial n_x \partial n_y} \delta\mu(\bar{x}) dS_y dS_x \quad (20)$$

After discretization using boundary elements, equation (18) becomes:

$$p = \{d\}^T \{\mu\} \quad (21)$$

Similarly, equation (20) becomes:

$$[Q]\{\mu\} = [H]\{v_b\} \quad (22)$$

Combining equations (21) and (22):

$$p = \{atv\}^T \{v_b\} \quad (23)$$

- 5 where  $\{atv\}$  is the Acoustic Transfer Vector, given by:

$$\{atv\}^T = \{d\}^T [Q]^{-1} [H] \quad (24)$$

When the acoustic pressure is evaluated at several locations, equation (23) can be rewritten as:

$$\{p\} = [ATM]^T \{v_b\} \quad (25)$$

- 10 where the Acoustic Transfer Matrix  $[ATM]$  is formed by the different Acoustic Transfer Vectors.

#### *Finite/infinite element method (FEM/IFEM) formulation*

- The solution of the Helmholtz equation based on a finite/infinite element  
15 approach leads to a system of linear equations:

$$([K] + j\rho\omega[D] - \omega^2[M])\{p\} = -j\rho\omega\{F\} \quad (26)$$

where K is the fluid stiffness matrix, D is the fluid damping matrix, M is the fluid mass matrix and F is the forcing vector, defined by:

$$\{F\} = [C]\{v_b\} \quad (27)$$

- 20 where C is a coupling matrix.

Combining equations (26) and (27), ):

$$\{p\} = [ATM]^T \{v_b\} \quad (28)$$

where the Acoustic Transfer Matrix  $[ATM]$  is given by:

$$[ATM]^T = \frac{-j\rho\omega[C]}{([K] + j\rho\omega[D] - \omega^2[M])} \quad (29)$$

- 25 Finally, for each finite element grid point, the acoustic pressure can be written as:

$$p = \{atv\}^T \{v_b\} \quad (30)$$

where  $\{atv\}$  is the Acoustic Transfer Vector, given by the corresponding row of

the  $[ATM]^T$  matrix.

### *Use of the Reciprocity Principle*

See F. Fahy, "The Reciprocity Principle and Applications in Vibro-Acoustics", Second International Congress on Recent Developments in air- and Structure Borne Sound and Vibration, 4-6 March 1992, 591-598 and/or K. R. Fyfe, L. Cremers, P. Sas, G. Creemers, "The use of acoustic streamlines and reciprocity methods in Automotive design sensitivity studies", Mechanical Systems and Signal Processing (1991) 5 (5), 431-441.

10 An aspect of the present invention is the use of the reciprocity principle for efficiently computing the Acoustic Transfer Vectors. As shown in equations (10), (15), (24) and (29), the evaluation of any field point related ATV requires the inversion of a matrix, whose order is approximately equal to the number of nodes of the discretized geometry (mesh).

15 The numerical evaluation of the inverse of an order  $n$  matrix is performed by solving a system of linear equations, whose right-hand-side is built from the unity matrix, leading typically to one matrix factorisation, and  $n$  back-substitution steps.

$$[A][X] = [I] \quad (31)$$

20 For large matrix orders, the computational effort and time required for performing the  $n$  back-substitution steps is very significantly larger than the factorisation time.

Looking to equation (10), it can be deduced that an ATV is a vector for which each coordinate corresponds to the pressure at the corresponding field point due to a unit excitation (vibrating velocity) at one point/element on the surface, and no excitation elsewhere.

$$atv_i = \{atv\} \begin{Bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{Bmatrix} \quad (32)$$

This is represented graphically in Fig. 4.

According to the reciprocity principle, the pressure response at a first location due to an excitation at a second location is strictly equal to the pressure response at the second location due to the same excitation at the first location. Therefore, if a source of strength  $Q_1$  at position 1 causes a pressure  $p_2$  at position 2, and if a source of strength  $Q_2$  at position 2 causes a pressure  $p_1$  at position 1, resulting in the following relationship:

$$p_1 Q_1 = p_2 Q_2 \quad \text{or} \quad \frac{p_1}{Q_2} = \frac{p_2}{Q_1} \quad (33)$$

Using the reciprocity principle to evaluate the ATV related to a specific field point means that a monopole source (point source) of strength  $Q_1$  should be positioned at the field point location 1, and the pressure is calculated on the boundary  $p_2$ . This is represented in figure 5.

Equation 11 shows how the ATV will be exploited to convert the boundary surface velocity (representing vibrations) into the field point acoustic pressure. Since a structure may be complex, containing T-junctions and other discontinuities, surface vibrations are better defined on surfaces, i.e. on finite elements, rather than at grid points (where the normal directly is not always uniquely defined). Therefore, it is required to define element-based acoustic transfer vectors.

In the case where the source placed on the boundary (in 2) is distributed over a small surface, equation (33) may be rewritten as:

$$p_1 Q_1 = \int_S p_2 v_2 dS \quad (34)$$

where  $v_2$  is the source velocity. In the particular case where the source is distributed over a single element, the pressure at any point on the surface can be approximated as:

$$p_2 = \langle p_2 \rangle \{N\} \quad (35)$$

where  $\{N\}$  is the interpolation function and  $\langle p_2 \rangle$  is the pressure evaluated at the element nodes. Similarly the source velocity can be approximated as:

$$v_2 = \langle N \rangle \{v_2\} \quad (36)$$

where  $\{v_2\}$  is the source velocity at the element nodes. Equations (34) to (36)

lead to the following relation:

$$p_1 Q_1 = \langle p_2 \rangle [C_e] \{v_2\} \quad (37)$$

where  $[C_e]$  is the element coupling matrix defined as:

$$[C_e] = \int_{S_e} \{N\} \langle N \rangle dS \quad (38)$$

- 5 For a monopole source, the source strength is given by:

$$Q_1 = \frac{4\pi}{j\rho\omega} A \quad (39)$$

where  $A$  is the monopole source amplitude. Combining (37) and (39):

$$p_1 = \frac{j\rho\omega}{4\pi A} \langle p_2 \rangle [C_e] \{v_2\} \quad (40)$$

which can be rewritten as:

$$10 \quad p_1 = [ATV_e] \{v_2\} \quad (41)$$

where  $[ATV_e]$  is the element-based acoustic transfer vector, computed by:

$$[ATV_e] = \frac{j\rho\omega}{4\pi A} \langle p_2 \rangle [C_e] \quad (42)$$

Applying at the field point a monopole source with a unit amplitude ( $A=1$  and  $\omega=2\pi f$ ), we finally obtain the following expression for evaluating the element-

- 15 based acoustic transfer vector:

$$[ATV_e] = \frac{j\rho f}{2} \langle p_2 \rangle [C_e] \quad (43)$$

showing that the ATV related to a given field point can then be computed from the boundary sound pressure values, correctly scaled.

- 20 The use of the reciprocity principle for evaluating the ATV's leads to two fundamental advantages:

1. The ATV's can be directly computed on an element basis, removing the ambiguity inherent to nodal values, in case of sharp edges and T-junctions, where the normal vector is not uniquely defined.
- 25 2. The ATV related to a given field point can then be computed with a single excitation vector, representing the monopole source excitation at the

field point. This leads to a single back substitution step. Since the number of field points (microphone locations) is usually several orders of magnitude lower than the number of boundary nodes, this results in tremendous gains in computational effort.

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### *Interpolation technique*

In the case of sound wave propagation in an open space, the fluid domain does not exhibit any resonant behaviour. The sound field is fully determined by various wave mechanisms, such as reflection by solid objects, absorption on damping surfaces, diffraction around edge and wave interference. Therefore, for non-resonant media, Acoustic Transfer Vectors are rather smooth functions of the frequency, and ATV coefficients can be accurately evaluated at any intermediate frequency, called a *slave* frequency, using a mathematical interpolation scheme, based on a discrete number of frequencies, called *master* frequencies. It is important to note that the structural vibrations cannot be similarly interpolated, since these are directly dependent on the highly resonant dynamic behaviour of any structure (vibration mode shapes).

20 At least two interpolation mechanisms can be used:

1. A polynomial interpolation scheme such as a linear interpolation scheme (see figure 6);
- 25 2. A spline interpolation scheme such as a cubic spline scheme (see figure 7), which is particularly well suited for approximating ATV's (De Boor, C. A Practical Guide to Splines, Springer-Verlag, New York, 1978).

Other interpolation schemes can be used , e.g. stochastic interpolation (Kriging interpolation) .

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The use of an interpolation scheme for evaluating the ATV's leads to

the following fundamental advantages:

1. It allows to build a complete frequency dependant ATV, with a very limited number of frequency lines to be computed. In order to build a complete frequency response with a traditional approach, the number of frequency lines to be computed is usually over a hundred. Using an interpolation mechanism, only a few master frequency lines needs to be explicitly computed, any other frequency being interpolated. The gain in computing time is usually over an order of magnitude.
2. Similarly, the ATV's have to be stored in a file database in order to be further exploited. For instance they may be stored in non-volatile memory of a computer system such as a disk or in mass storage or for example on a CD-ROM. The interpolation mechanism allows storage of only the master frequency ATV's in the database. The corresponding vector at any intermediate frequency may then be directly evaluated at a neglectible processing cost, and therefore does not need to be stored in the database. The gain in disk space is usually over an order of magnitude.
3. For some specific applications, such as automotive powertrains, the acoustic performance has to be evaluated for many different operational conditions (e.g. different rpm's), each of these having a different frequency contents. Using the ATV interpolation mechanism, the same set of master frequencies can be used to build a unique basis of ATV's, which are then interpolated and combined with the structural response corresponding to any load case, at any intermediate frequency needed for the specific operational condition.

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#### *Inverse Numerical Acoustics*

Equations (11) and (25) are the basic relations for the inverse numerical acoustics theory. The sound pressure is measured at a large number of microphone (field points) and the boundary velocity can be obtained using the relationship:

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$$\{v_b\} = [ATM]^{-1} \{p\} \quad (44)$$

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(45)

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From equations (45) and (46) it follows:

(47)

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### Panel Contribution Analysis

Panel Contribution Analysis is a fundamental engineering tool to guide



the product refinement in the development process. It is shown schematically in Fig. 8 in which contributions reaching the ear of person in a vehicle come from different parts or panels of the vehicle. Equations (9), (16), (23) or (30) can be rewritten as:

$$p = \sum_{e=1}^{ne} p_e = \sum_{e=1}^{ne} [ATV_e]^T \{v_e\} \quad (48)$$

where  $ne$  is the total number of elements. Subsequently, the contribution of a panel is given by the summation of element contributions over the panel elements:

$$p_c = \sum_{pe} p_e = \sum_{pe} [ATV_e]^T \{v_e\} \quad (49)$$

where  $pe$  stands for panel elements.

#### Modal ATV

The engineering process to compute the structural velocity  $\{v_b\}$  on a vibrating surface relies usually on the structural finite element method, and often on a modal superposition approach, where the structural response is expressed as a linear combination of the mode shapes of the body:

$$\{v_b\} = [\Phi_n] \{mpf\} \quad (50)$$

Where  $[\Phi_n]$  is the matrix composed of the modal vectors, projected on the normal direction to the boundary surface, and  $\{mpf\}$  are the modal participation factors, also called the modal response of the structural model, at a given excitation frequency. The term "mode" relates not only to natural or "eigen-" frequencies but also to operational frequencies such as defined by Ritz vectors.

Combining (50) with Equations (9), (16), (23) or (30) leads to:

$$p = \{atv\}^T [\Phi_n] \{mpf\} \quad (51)$$

where

$$\{atv\}^T [\Phi_n] = \{matv\}^T \quad (52)$$

is called the Modal Acoustic Transfer Vector, which can be directly combined

with the modal response to give the sound pressure at a field point:

$$p = \{\text{matv}\}^T \{\text{mpf}\} \quad (53)$$

The concept of Modal Acoustic Transfer Vectors can be extended to any possible deformation shapes of the body, so-called Ritz vectors. This is especially interesting in the case of inverse numerical acoustics, where the unknown becomes then the modal participation factors.

### *Optimisation strategy*

An embodiment of the present invention includes a method that allows the numerical acoustic radiation prediction calculation to be extremely fast, the main computational intensive task (calculation of the ATV's) being done as a pre-processing step. Therefore, it is directly possible and practical to integrate the acoustic radiation prediction within an optimisation loop, where the objective function is the acoustic performance (sound pressure levels, or radiated power according to the ISO3744-1981 procedure), and where the design variables are either structural or acoustic design variables.

An optimisation process in accordance with an embodiment of the present invention is shown at figure 9a. The optimisation process will normally be carried out on a digital computing system. It will typically combine a structural dynamic finite element solver (step 104) together with an ATV-based acoustic prediction tool (step 106). In step 102 a set of ATV's is evaluated for the structure to be analysed using, for example, the reciprocity principle as described above. In step 104 a solver is used to determine the surface vibration velocities of the structure. Commercially available solvers are, for example, MSC.Nastran from MSC Software, USA, ANSYS from Ansys Inc. USA, ABAQUS from HKS Inc. In step 106 the ATV's are combined with the surface velocities to determine an acoustic signature at one or more field points. As part of this process an ATV interpolation step (108) may optionally be used to increase the number of available ATV's.

The achieved performance is compared in step 110 with a desired performance or with a previously calculated performance to determine if a

desired or optimum performance is achieved. If not, a design change can be made in step 112, e.g. add or remove stiffening elements to the structure, change a dimension of the structure or a material (e.g. to provide more or less damping). Provided these changes are relatively minor there will be no appreciable change in the ATV's and therefore step 102 does not need to be repeated. Instead the vibration response is recalculated in step 104 and a new performance analysis carried out. Avoiding the re-calculation of the ATV's is a significant advantage of this embodiment.

A similar methodology is shown in Fig. 9b using MATV's. In this case step 106 is replaced with a step 116 of obtaining the MATV's by projecting the ATV's determined in step 102 into a co-ordinate system defined by the vibrational modes determined in step 114. The modes used may be natural modes, e.g. the shapes of the structure determined by the natural or eigen-frequencies of the structure, or the modes may be operational modes, e.g. those determined by Ritz vectors. An optional interpolation step 118 may be used to generate MATV's at intermediate frequencies. The response at one or more field points is calculated from the MATV's in step 120 in which the results are compared with a previously calculated result or with a desired result. If unsatisfactory a design change may be made in step 122 and re-evaluated in steps 114 – 120.

### *Implementation*

The methods of the present invention may be implemented on a processing engine such as a workstation or a personal computer. The processing engine may be a server accessible via a telecommunications network such as a LAN, a WAN, the Internet, an Intranet. The server may be adapted to carry out any of the methods of the present invention, for example a descriptor file of a body may be entered at a near terminal and transmitted to the server via the Internet. The server then carries out one of the methods of the present invention and returns to a near location e.g. an e-mail box, any response of any of the methods in accordance with the present invention.

Such a response may be, for instance, an ATV, a vibrational amplitude such as an acoustic pressure level, a surface vibration of the body, a revised design of at least a part of the body.

By way of example only and with reference to Figure 10, in one embodiment a near terminal 12 which communicates with a server 14 via a communications network 16 such as the Internet, or an Intranet. Information about a body to be analyzed or assessed is entered into an input 18 at the near terminal 12, e.g. in the form of a representation of the body and coordinates of a reference point. This is communicated to the server 14 which performs one or more methods of the invention and provides to an output 20 of the near terminal 12 a response in accordance with the methods described above. The server 14 operates in conjunction with a computer program which implements the invention and which is stored on a data carrier 22 which is accessible to the server 14.

The methods of the present invention may be implemented as computer programs which may be stored on data carriers such as diskettes or CD-ROMS. These programs may also be downloaded via the Internet or any other telecommunications network.

Fig. 11 is a schematic representation of a computing system which can be utilized with the methods and in a system according to the present invention. A computer 10 is depicted which may include a video display terminal 14, a data input means such as a keyboard 16, and a graphic user interface indicating means such as a mouse 18. Computer 10 may be implemented as a general purpose computer, e.g. a UNIX workstation.

Computer 10 includes a Central Processing Unit ("CPU") 15, such as a conventional microprocessor of which a Pentium III processor supplied by Intel Corp. USA is only an example, and a number of other units interconnected via system bus 22. The computer 10 includes at least one memory. Memory may include any of a variety of data storage devices known to the skilled person such as random-access memory ("RAM"), read-only memory ("ROM"), non-volatile read/write memory such as a hard disc as known to the skilled person.

For example, computer 10 may further include random-access memory ("RAM") 24, read-only memory ("ROM") 26, as well as an optional display adapter 27 for connecting system bus 22 to an optional video display terminal 14, and an optional input/output (I/O) adapter 29 for connecting peripheral devices (e.g., disk and tape drives 23) to system bus 22. Video display terminal 14 can be the visual output of computer 10, which can be any suitable display device such as a CRT-based video display well-known in the art of computer hardware. However, with a portable or notebook-based computer, video display terminal 14 can be replaced with a LCD-based or a gas plasma-based flat-panel display. Computer 10 further includes user interface adapter 19 for connecting a keyboard 16, mouse 18, optional speaker 36, as well as allowing optional physical value inputs from physical value capture devices such as sensors 40 of an external system 20. The sensors 40 may be any suitable sensors for capturing physical parameters of system 20. These sensors may include any sensor for capturing relevant physical values required for solution of the problems addressed by the present invention. Such sensors can be microphones and accelerometers (for acoustics). Additional or alternative sensors 41 for capturing physical parameters of an additional or alternative physical system 21 may also be connected to bus 22 via a communication adapter 39 connecting computer 10 to a data network such as the Internet, an Intranet, a Local or Wide Area network (LAN or WAN) or a CAN. This allows transmission of physical values or a representation of the physical system to be simulated over a telecommunications network, e.g. entering a description of a physical system at a near location and transmitting it to a remote location, e.g. via the Internet, where a processor carries out a method in accordance with the present invention and returns a parameter relating to the physical system to a near location.

The terms "physical value capture device" or "sensor" includes devices which provide values of parameters of a physical system to be simulated. Similarly, physical value capture devices or sensors may include devices for transmitting details of evolving physical systems. The present invention also

includes within its scope that the relevant physical values are input directly into the computer using the keyboard 16 or from storage devices such as 23.

A parameter control unit 37 of system 20 and/or 21 may also be connected via a communications adapter 38. Parameter control unit 37 may receive an output value from computer 10 running a computer program for numerical analysis in accordance with the present invention or a value representing or derived from such an output value and may be adapted to alter a parameter of physical system 20 and/or system 21 in response to receipt of the output value from computer 10. For example, a dimension of the body or structure to be simulated may be altered, a material may be changed, a stiffening or dampening element may be added or removed.

Computer 10 also includes a graphical user interface that resides within machine-readable media to direct the operation of computer 10. Any suitable machine-readable media may retain the graphical user interface, such as a random access memory (RAM) 24, a read-only memory (ROM) 26, a magnetic diskette, magnetic tape, or optical disk (the last three being located in disk and tape drives 23). Any suitable operating system and associated graphical user interface (e.g., Microsoft Windows) may direct CPU 15. In addition, computer 10 includes a control program 51 which resides within computer memory storage 52. Control program 51 contains instructions that when executed on CPU 15 carry out the operations described with respect to any of the methods of the present invention.

Those skilled in the art will appreciate that the hardware represented in FIG. 11 may vary for specific applications. For example, other peripheral devices such as optical disk media, audio adapters, or chip programming devices, such as PAL or EPROM programming devices well-known in the art of computer hardware, and the like may be utilized in addition to or in place of the hardware already described.

In the example depicted in Fig. 11, the computer program product (i.e. control program 51) can reside in computer storage 52. However, it is important that while the present invention has been, and will continue to be,

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